

NAG Toolbox for MATLAB

f07ff

1 Purpose

f07ff computes a diagonal scaling matrix S intended to equilibrate a real n by n symmetric positive-definite matrix A and reduce its condition number.

2 Syntax

```
[s, scond, amax, info] = f07ff(a, 'n', n)
```

3 Description

f07ff computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B , $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham 2002).

4 References

Higham N J 2002 *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The matrix A whose scaling factors are to be computed. Only the diagonal elements of the array **a** are referenced.

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **a**.

n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **s(*)** – double array

Note: the dimension of the array **s** must be at least $\max(1, \mathbf{n})$.

If **info** = 0 on exit, **s** contains the diagonal elements of the scaling matrix *S*.

2: **scond** – double scalar

If **info** = 0 on exit, **scond** contains the ratio of the smallest value of $\mathbf{s}(i)$ to the largest value of $\mathbf{s}(i)$.

If **scond** ≥ 0.1 and **amax** is neither too large nor too small, it is not worth scaling by *S*.

3: **amax** – double scalar

$\max |a_{ij}|$. If **amax** is very close to overflow or underflow, the matrix *A* should be scaled.

4: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter *i* had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **a**, 3: **lda**, 4: **s**, 5: **scond**, 6: **amax**, 7: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = *i*, the *i*th diagonal element of *A* is not positive (and hence *A* cannot be positive-definite).

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Further Comments

The complex analogue of this function is f07ft.

9 Example

```
a = [4.16, -312000, 0.56, -0.1;
      0, 50300000000, -83000, 118000;
      0, 0, 0.76, 0.34;
      0, 0, 0, 1.18];
[s, acond, amax, info] = f07ff(a)

s =
    0.4903
    0.0000
    1.1471
    0.9206
acond =
    3.8871e-06
```

```
amax =  
      5.0300e+10  
info =  
          0
```
